

Quintessence in Brane Cosmology

Pedro F. González-Díaz.

Isaac Newton Institute, 20 Clarkson Road, Cambridge, CB3 0EH, UK
and

Instituto de Matemáticas y Física Fundamental
Consejo Superior de Investigaciones Científicas
Serrano 121, 28006 Madrid, SPAIN

March 3, 2000

Abstract

In order to reconcile the non conventional character of brane cosmology with standard Friedmann cosmology, we introduce in this paper a slowly-varying quintessence scalar field in the brane and analyse the cosmological solutions corresponding to some equations of state for the scalar field. Different compensation mechanisms between the cosmological constant in the bulk and the constant tension resulting from the combined effect of ordinary matter and the quintessence scalar field are derived or assumed. It has been checked that the Randall-Sundrum approach is not necessarily the best procedure to reconcile brane and standard cosmologies, and that there exists at least another compensating mechanism that reproduces a rather conventional behaviour for an accelerating universe.

In order to reconcile the mismatch of the scales of particle physics and gravity it was recently suggested [1] that the latter scale can be lowered all the way to the weak scale by introducing large extra dimensions, so opening up the possibility for new primordial cosmological scenarios. Randall and Sundrum later proposed [2] that the size of such extra dimensions could still be kept small, with the background metric being a non flat slice of anti-de Sitter space due to the existence of a negative bulk cosmological constant which is exactly balanced by the tensions on the two branes occurring in this scenario. It is the so-generated curved character of the space-time which causes the physical scales on the two branes to take on exponentially different values. This has prompted a lot of activity [3] on the possibility that we live just in a three-dimensional one-brane world embedded in a higher dimensional space, while gravity pervades the whole highest dimensional space which, contrary to the Kaluza-Klein spirit, need not even be compact according to the Randall-Sundrum philosophy. The cosmological evolution of the brane universe has already been extensively investigated by many authors [4-8].

It was however shown by Binétruy, Deffayet, Ellwanger and Langlois [4] that the evolution of a brane-like universe is not viable in the sense that the cosmological field equations corresponding to it do not match the analogous Friedmann-Robertson-Walker equations of standard cosmology, the essential difference being that the energy density in the brane appears quadratically, rather than linearly, in the right hand side of the Einstein equations. It was later noted [5-7] that brane and standard cosmologies could still be reconciled by again using the Randall-Sundrum approach [2], so that a negative cosmological constant is introduced in the bulk which is exactly compensated by a constant tension in the brane in such a way that the nonlinear term for the energy density in the field equations would nearly vanish (leaving the usual Friedmann-Robertson-Walker evolution) at late times, and is relevant only at the earliest times, where cosmology becomes highly non conventional. Thus, much as quantum effects are currently thought to remarkably modify Einstein general relativity only at the initial Planck era, one could also regard the relevant primordial deviations from standard cosmology in the brane to be caused by some sort of topological effects which are only relevant in the realm of quantum gravity.

It is the aim of the present work to discuss and generalize the above interpretation by obtaining particular exact solutions to the five-dimensional Einstein equations for a brane universe which corresponds to an observable ordinary matter in the brane and a negative cosmological constant in the bulk that can also now be compensated by the combined effect of the constant ordinary-matter tension and a vacuum quintessence scalar-field tension [9] in the brane. This cancellation mechanism so as the one derived from the Randall-Sundrum condition, comes in our model quite naturally from the constraint on the quintessence potential which is derived from the field equations and conservation laws (or as particular *ansätze* from that potential in the case of constant scalar field). We note that such a compensating mechanism actually generalizes the Randall-Sundrum paradigm which in this paper will be extended to encompass not just a vacuum constant brane tension, but also the tension derived from the observable ordinary matter. Among the cosmological models that we have found (which include

accelerating and decelerating open or closed universes with an initial non conventional phase), there are some "exotic" universes (that is universes with negative energy density for ordinary matter) which correspond to particular solutions that do not contain any non conventional initial evolution.

According to Binétruy et al. [4,5], in a five-dimensional space-time the generalized time-time component of the Friedmann equations can be written on a three-brane as

$$\frac{\dot{R}^2}{R^2} = \frac{\kappa^2}{6}\rho_B + \frac{\kappa^4}{36}\rho_b^2 + \frac{C}{R^4} - \frac{k}{R^2}, \quad (1)$$

where the overhead dot means derivative with respect to time, R is the scale factor in the brane, ρ_B and ρ_b are the energy densities in the bulk and the brane, respectively, C is an integration constant which is probably related to the choice of the initial conditions for the universe and can be interpreted as an effective radiation term [4,5], k is the topological curvature ($k = 0, \pm 1$), and

$$\kappa^2 = 8\pi G_{(5)} \equiv M_{(5)}^{-3}, \quad (2)$$

with $G_{(5)}$ the five-dimensional Newton constant and $M_{(5)}$ the five-dimensional reduced Planck mass. In what follows we shall restrict ourselves to the flat case ($k = 0$) and assume that the boundary conditions for the universe are such that $C = 0$. The nonlinear term proportional to ρ_b^2 in Eq. (1) makes the cosmology resulting from this equation highly non conventional. As pointed out above, several authors [3,5-7] have suggested that the discrepancy between brane and standard cosmologies may be greatly alleviated by introducing a constant tension in the brane universe which compensates the negative cosmological constant in the bulk, within the spirit of the Randall-Sundrum approach. On the other hand, the conservation law for the energy in the brane which is compatible with Eq. (1) can be written as [5]

$$\dot{\rho}_b + 3\frac{\dot{R}}{R}(\rho_b + p_b) = 0, \quad (3)$$

with p_b the pressure in the brane. We shall furthermore take $p_M = 0$ for the state equation of the ordinary fluid in the brane.

If, instead of a constant tension term, we introduce in the brane a vacuum scalar quintessence field ϕ , with state equation [9]

$$p_\phi = \omega\rho_\phi, \quad 0 \leq \omega < -1 \quad (4)$$

(with $\omega = -1$ corresponding to the constant tension case), which behaves like a perfect fluid, then we have for the energy density in the brane,

$$\rho_b = \rho_M + \rho_\phi \quad (5)$$

(with ρ_M the energy density for the ordinary matter), so that Eq. (3) becomes

$$\dot{\rho}_b + 3\frac{\dot{R}}{R}[\rho_M + \rho_b(1 + \omega)] = 0. \quad (6)$$

The quintessence field ϕ is assumed to be a slowly varying scalar field which can be defined by

$$\kappa^2 \rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi) > 0, \quad \kappa^2 p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi) \leq 0,$$

where $V(\phi)$ is the potential energy for the field ϕ . Furthermore, since no interaction between the scalar field ϕ and ordinary matter M is assumed to occur in the brane, one can take independent conservation laws for M and ϕ [10]

$$\rho_M = \rho_{M0} \left(\frac{R_0}{R} \right)^3, \quad \rho_\phi = \rho_{\phi 0} \left(\frac{R_0}{R} \right)^{3(1+\omega)}, \quad (7)$$

with the subscript 0 denoting current value. Using then the second of Eqs. (7), one can integrate Eq. (6) to yield

$$R^3 \rho_b - \frac{R_0^{3(1+\omega)} \rho_{\phi 0}}{R^{3\omega}} = D, \quad (8)$$

where D is an integration constant that gives the total mass in the brane universe, $D \equiv M$. Replacing the expression for ρ_b obtained from Eq. (8) in the field equation (1) for $k = 0$ and $C = 0$, we finally get a differential constraint on the scale factor R which must be satisfied by all possible solutions:

$$\dot{R}^2 = \frac{\kappa^2}{6} \rho_B R^2 + \frac{\kappa^4}{36} \left[\frac{M}{R^2} + \frac{\rho_{\phi 0} R_0^{3(1+\omega)}}{R^{3\omega+2}} \right]^2. \quad (9)$$

In order to set a suitable compensating mechanism able to alleviate or solve the discrepancy between brane and standard cosmologies and, in particular, to check whether the Randall-Sundrum approach or other possible conditions can be successfully applied with that purpose in the case $\omega = -1$, we shall derive from Eq. (9) another constraint, that on the scalar-field potential $V(\phi)$ satisfying the field equation (1) and all conservation laws (7) and (8). From these expressions and the definition of the field ϕ , first one obtains

$$\frac{R}{R_0} = \left(\frac{V}{V_0} \right)^{-\frac{1}{3(1+\omega)}}, \quad \frac{\dot{R}}{R_0 H_0} = \left(\frac{V}{V_0} \right)^{-\frac{3\omega+5}{6(1+\omega)}} \frac{V'}{V_0'}, \quad (10)$$

where H_0 is the current value of the Hubble constant, $' = d/d\phi$, and [10]

$$V_0 = \frac{1-\omega}{2} \kappa^2 \phi_{\phi 0}, \quad V_0' = \pm \frac{3H_0(1-\omega)}{2} \sqrt{(1+\omega) \kappa^2 \rho_{\phi 0}}. \quad (11)$$

From expressions (10) and (11), Eq. (9) can now be directly transformed into the wanted constraint. It is nevertheless convenient first introducing the dimensionless cosmological parameters

$$\Omega_B = \frac{\kappa^2 \rho_B}{3H_0^4}, \quad \Omega_M = \frac{\kappa^2 \rho_{M0}}{3H_0^2}, \quad \Omega_\phi = \frac{\kappa^2 \rho_{\phi 0}}{3H_0^2}. \quad (12)$$

We then obtain for the constraint on the scalar potential:

$$\frac{V'^2}{H_0^2 V_0^2} = \frac{\Omega_B}{2} \left(\frac{V}{V_0} \right) + \frac{1}{4} \left[\Omega_M \left(\frac{V}{V_0} \right)^{\frac{\omega+3}{2(1+\omega)}} + \Omega_\phi \left(\frac{V}{V_0} \right)^{\frac{3}{2}} \right]^2. \quad (13)$$

This expression has been derived by implicitly assuming that $\omega \neq -1$. For $\omega = -1$, $V = V_0 \neq 0$ and $\phi = \phi_0 \neq 0$ are both constant, and $V'_0 = 0$. Thus, in the case that the field ϕ reduces to a cosmological constant Λ in the brane, we attain that the quantity

$$I \equiv \Omega_B + \frac{1}{2} (\Omega_M + \Omega_\Lambda)^2$$

(or $I = \Omega_B + 1/2$ if we use the triangular cosmological constraint $\Omega_k + \Omega_M + \Omega_\Lambda = 1$ on the brane) is mathematically indeterminate. One may then employ different *ansätze* for the value of I . Three of such possible conditions will appear to be particularly interesting. If we set $I = \Omega_M^2/2$, the Randall-Sundrum condition $\Omega_B = -\Omega_\Lambda^2/2$ is recovered, if we set $I = 0$, it follows that $\Omega_B = -1/2$, and finally if $I = 1/2$, then $\Omega_B = 0$. All of these three *ansätze* will be used later on this paper.

From Eqs. (10) we can now derive the expression

$$\phi - \phi_0 = -3(1 + \omega) \frac{V_0 H_0}{V'_0} \int dt \left(\frac{R}{R_0} \right)^{-\frac{3}{2}(1+\omega)}, \quad (14)$$

which will be useful in what follows as well. For quintessence, the most interesting models are those where ω is not a constant and are defined in terms of an inverse-power law potential for the field ϕ [11]. These models improve the fine-tuning problem associated with the cosmological constant [11], they solve the cosmic coincidence problem [12], and they can be implemented in the realm of high energy physics [13]. Actually, one can obtain solutions to constraint (13) having the form of an inverse-power law, for a generic ω , only if particular values for the cosmological parameters Ω 's are assumed (for example, if we set $\Omega_B = \Omega_M = 0$, then the potential that satisfies (13) has the form $V \propto \phi^{-2}$), but one cannot obtain analytical solutions in closed form for generic values of ω and the Ω 's. Therefore, in the remaining of this paper we shall consider different solutions to the constraint equations on V and R for given particular values of the quintessence parameter ω [9], and discuss their physical motivation in the relevant cases. Let us start with $\omega = 0$ for which case the solution to constraint (9) is

$$R(t) = R_0 \left[\sinh \left(\sqrt{\frac{9H_0^2 \Omega_B}{2}} t \right) \right]^{\frac{1}{3}}. \quad (15)$$

Similarly to how it happens for the solution that corresponds to the case in which there are neither a cosmological constant nor a quintessence field in the brane, the scale factor (15) clearly has a nonconventional behaviour at early times, $R \propto t^{1/3}$, followed by an exponential expansion at any late times. Inserting the expression of R given by Eq. (15) in the general expression (14) and using Eqs. (11) for $\omega = 0$, we have

$$\phi - \phi_0 = \pm \sqrt{\frac{2\Omega_\phi}{3\Omega_B}} F \left[\arccos \left(\frac{1 - \sinh(a_0 t)}{1 + \sinh(a_0 t)} \right), \frac{1}{\sqrt{2}} \right], \quad (16)$$

where F denotes the elliptic integral of the first kind [14], and

$$a_0 = \sqrt{\frac{9H_0^2\Omega_B}{2}}; \quad (17)$$

hence it follows

$$\sinh(a_0 t) = \frac{1 - \operatorname{cn}\left[\sqrt{\frac{3\Omega_B}{2\Omega_\phi}}(\phi - \phi_0)\right]}{1 + \operatorname{cn}\left[\sqrt{\frac{3\Omega_B}{2\Omega_\phi}}(\phi - \phi_0)\right]}, \quad (18)$$

in which cn is a Jacobian elliptic function [15]. Using then the first of the Eqs. (10) we obtain for the scalar potential

$$V(\phi) = \frac{V_0}{\sinh(a_0 t)} = V_0 \left\{ \frac{1 + \operatorname{cn}\left[\sqrt{\frac{3\Omega_B}{2\Omega_\phi}}(\phi - \phi_0)\right]}{1 - \operatorname{cn}\left[\sqrt{\frac{3\Omega_B}{2\Omega_\phi}}(\phi - \phi_0)\right]} \right\}. \quad (19)$$

It can be easily seen that the scalar potential (19) satisfies constraint (13), provided $\Omega_M + \Omega_\phi = 1$ and $\Omega_B = 1/2$. The latter condition implies a positive cosmological constant in the bulk and corresponds to an ansatz $I = 1$, so that neither the Randall-Sungrum approach nor any of the other two above-alluded conditions can hold in this case. Clearly, if we set the initial conditions immediately after inflation, i.e. at a redshift $z \sim 10^{28}$, then $\phi - \phi_0 \propto F\left[\arccos(z/(z+2)), 1/\sqrt{2}\right]$, which is very small initially and along most of its evolution, except when it approaches current values where $\phi - \phi_0 \sim 1$, so providing a reason why the quintessential field starts dominating only now, and hence solving the cosmic coincidence problem [13]. On the other hand, for most of its cosmological evolution, potential (19) can be approximated as $V + V_0 \propto V_0(\phi - \phi_0)^{-2}$, i.e. an inverse-power law potential which might be linked to particle physics models [13].

We consider next the ansatz $I = 1/2$ (i.e. $\Omega_B = 0$) for the case that the quintessence state equation takes the particular expression corresponding to $\omega = -1/2$. We distinguish the approximate early and late time solutions:

$$R(t) = R_0 \left(\frac{3}{2} \Omega_M H_0 t \right)^{\frac{1}{3}} \quad (20)$$

for early times, and

$$R(t) = R_0 \left(\frac{3}{4} \Omega_\phi H_0 t \right)^{\frac{2}{3}} \quad (21)$$

for late times. It is worth noticing that the same qualitative behaviour for the scale factor as that given by solutions (20) and (21) (that is, an initial non conventional expansion, followed by a conventional one up to arbitrarily large time) was also obtained using the Randall-Sungrum approach with a cosmological constant in the brane [5]. Following then the same steps as for $\omega = 0$, we finally get again in this case an inverse-power law for the scalar potential; i.e.

$$V(\phi) \propto (\phi - \phi_0)^{-2}, \quad (22)$$

with the proportionality constants being given by simple functions of Ω_ϕ and Ω_M , both for early and late times. It can be checked that this potential satisfies constraint (13) by simply imposing the cosmological triangular condition $\Omega_k + \Omega_\phi + \Omega_M = 1$ in the brane. Potential (22) may once again be implemented in the realm of high energy physics and from it and the first of Eqs. (10), one can deduce that the quintessential field becomes in this case proportional to $(1+z)^{-3/4}$, so that one can also solve the cosmic coincidence problem as well.

We finally come to the case $\omega = -1$. As pointed out above, this corresponds to having a cosmological constant Λ in the brane, so that $V = V_0$, $\phi = \phi_0$ and I becomes indeterminate. Three ansätze will be considered for I : $I = \Omega_M^2/2$ (corresponding to the Randall-Sundrum approach), $I = 0$ and $I = 1/2$. These particular approaches are associated with values of the cosmological constant in the bulk given by $\Omega_B = -\Omega_\Lambda^2/2$, $\Omega_B = -1/2$ and $\Omega_B = 0$, respectively. For the first of these conditions, the solution reads:

$$R(t) = R_0 \left[\frac{9H_0^4 \Omega_M \Omega_\Lambda}{8} \left(t^2 + \frac{4t}{3H_0^2 \Omega_\Lambda} \right) \right]^{\frac{1}{3}}, \quad (23)$$

which corresponds to the same qualitative behaviour as for the solution obtained when we take $I = 1/2$, $\omega = -1/2$, such as it was mentioned above. If we allow the quantity I to be in the close neighbourhood of the Randall-Sundrum value $\Omega_M^2/2$ and keep $\ell = I - \Omega_M^2/2$ nonzero but very small, then it is obtained for R

$$R(t) = R_0 \left\{ \frac{1}{2\ell^2 \kappa^4} \left[\left(R_0^{-3} - \frac{\Omega_M \Omega_\Lambda}{2} \right) \sinh(3\ell \kappa^2 t) + \frac{\Omega_M \Omega_\Lambda}{2} (\cosh(3\ell \kappa^2 t) - 1) \right] \right\}^{\frac{1}{3}}. \quad (24)$$

Solution (24) describes the evolution of a brane universe which initially expands non conventionally, $R \propto t^{1/3}$, to enter then the customary regime, $R \propto t^{2/3}$, and finally an exponential realm, $R \propto \exp(3\ell \kappa^2 t)$, that keep holding forever. We thus recover the solution first derived by Binétruy et al. [5] in the case of a general equation of state for the ordinary matter.

The use of the ansatz $\Omega_B = 0$ leads to a solution to the constraint on R which reads:

$$R(t) = R_0 \left\{ \frac{\Omega_M}{\Omega_\Lambda} \left[\exp \left(\frac{\kappa^2 \Omega_\Lambda}{R_0^3 \Omega_M} t \right) - 1 \right] \right\}^{\frac{1}{3}}, \quad (25)$$

which, for a convenient choice of the cosmological parameters Ω_M and Ω_Λ , shows a qualitative behaviour similar to that is predicted by solution (24).

Let us finally consider the case where $I = 0$, so that $\Omega_B = -1/2$. Then from the constraint on R we have for the cosmological time

$$t = - \int \frac{dx}{x \sqrt{M^2 x^2 + 2M \rho_\Lambda x + \left(\rho_\Lambda^2 - \frac{9H_0^4}{\kappa^4} \right)}}, \quad (26)$$

where $x = R^{-3}$. Now, we shall examine the three possible situations which appear depending on whether Ω_Λ^2 is larger, equal or smaller than unity. If $\Omega_\Lambda^2 > 1$ then we get

from Eq. (26):

$$R(t) = \frac{1}{(\Omega_\Lambda^2 - 1)^{\frac{1}{3}}} \left\{ \left(M^2 - \frac{\kappa^4}{36H_0^4} \right) \sinh \left(\frac{3}{2} \sqrt{\Omega_\Lambda^2 - 1} H_0^2 t \right) + \left(M^2 + \frac{\kappa^4}{36H_0^4} \right) \left[\cosh \left(\frac{3}{2} \sqrt{\Omega_\Lambda^2 - 1} H_0^2 t \right) - 1 \right] \right\}^{\frac{1}{3}}. \quad (27)$$

Now, if $M > \kappa^2/6H_0^2$ one should expect from solution (27) the same qualitative evolutive behaviour as that was obtained from solution (24), including the initial non conventional phase. More interesting are the cases where the mass of the brane universe is constrained to be $M = \pm \kappa^2/6H_0^2$. Then from the cosmological triangular condition on the brane we have $\Omega_\Lambda = 1 \mp \kappa^2/(18H_0^4 R_0^3)$. In these cases the scale factor (27) reduces to

$$R(t) = \left\{ \frac{2M^2}{\Omega_\Lambda^2 - 1} \left[\cosh \left(\frac{3}{2} \sqrt{\Omega_\Lambda^2 - 1} H_0^2 t \right) - 1 \right] \right\}^{\frac{1}{3}}, \quad (28)$$

which represents a universe which initially expands according to conventional cosmology ($R \propto t^{2/3}$) to finally grow exponentially, without passing through any non conventional $t^{1/3}$ -phase. Thus, if it turned finally out that the observable universe is actually accelerating [16], the brane solution (28) could be regarded as a good candidate to describe it. The price to be paid for getting such a conclusion is to allow for an universe with either a large positive mass $\Omega_M > 2$ or a negative energy density $\Omega_M < 0$.

If $\Omega_\Lambda^2 < 1$, then we obtain the closed solution

$$\frac{R(t)}{R_0} = \left\{ \frac{\Omega_M}{1 - \Omega_\Lambda^2} \left[\Omega_\Lambda + \sin \left(\frac{3}{2} h_0^2 \sqrt{1 - \Omega_\Lambda^2} t \right) \right] \right\}^{\frac{1}{3}} - \left(\frac{\Omega_M \Omega_\Lambda}{1 - \Omega_\Lambda^2} \right)^{\frac{1}{3}}, \quad (29)$$

which appears to be not quite fashionable not just for its initial non conventional behaviour, but mainly for its disconform to recent cosmological observations. More interesting appears to be the solution that corresponds to the case $\Omega_\Lambda^2 = 1$ which exactly coincides with the solution obtained when one imposes the Randall-Sungrum approach (Eq. (23)).

Before closing up, it seems interesting to notice an additional physical motivation for the general model used in this letter. If we include among the nongeometrical contributions to the energy density entering the definition of the luminosity distance D_L [17] the contribution from the bulk Ω_B , then using the definition of the redshift in terms of the scale factor and the first of Eqs. (10) we obtain for our flat model:

$$D_L \equiv D_L(\Omega_B) = \frac{V_0'(1+z)}{2H_0(\omega+1)V_0^{\frac{1}{2} + \frac{1}{3(\omega+1)}}} \int \phi(0)^{\phi(z)} \frac{d\phi}{V^{\frac{3\omega+1}{6(\omega+1)}}}, \quad (30)$$

with H_0 the Hubble constant. Bringing then this expression into the magnitude(m^{eff})-redshift(z) relation [16], our predictions can be compared with the results obtained in observations of distant supernovas. For inverse-power law potentials $V \propto \phi^{-\alpha}$, with

$\alpha > 0$, the use of Eq. (30) leads to magnitude-redshift plots which predict a suitable accelerating behaviour. An interesting physical consequence is that the shape of these plots is insensitive to the value of Ω_B , even though m^{eff} depends on Ω_B . It follows that one cannot extract any information about the bulk from supernova observations.

Clearly, there appear to be many other interesting solutions corresponding to other choices of quintessence parameter ω and/or the indeterminate quantity I when $\omega = -1$ which may be dealt with and interpreted following lines analogous to those considered in this paper. Moreover, one should check the stability of our ω -constant potentials to quantum corrections. However, the results obtained so far seem to be forcefull enough to draw off the following conclusion. When applied to brane cosmology, the Randall-Sungrum approach is nothing but just another more condition among a presumably large number of similar ansätze which can all alleviate or even solve the mismatch between brane and standard cosmology. It appears e.g. that for the case $\omega = -1$ there actually exists at least a particular condition, other than that of Randall and Sungrum, that is expressed as $\Omega_B = -1/2$, which may even lead to a cosmological model without any non conventional expansion phase. In one case this solution corresponds to a universe filled with matter having negative energy density. This situation would mean violation of the classical energy conditions [18] and could imply the existence of causality-violating processes involving superluminal travels or closed timelike curves [19].

Acknowledgements

The author thanks C.L. Sigüenza for useful comments and a careful reading of the manuscript. This work was supported by DGICYT under Research Project No. PB97-1218.

References

- 1 N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B429 (1998) 263; I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B436 (1998) 257.
- 2 L. Randall and R. Sundrum, Phys. Rev. Lett. 83 (1999) 3370; 4690.
- 3 C. Csáki, M. Graesser, L. Randall, and J. Terning, *Cosmology of Brane Models with Rodion Stabilization*, hep-ph/9911406, and references therein.
- 4 P. Binétruy, C. Deffayet, and D. Langlois, *Non-Conventional Cosmology from a Brane Universe*, hep-th/9905012.
- 5 P. Binétruy, C. Deffayet, U. Ellwanger, and D. Langlois, *Brane Cosmological Evolution in a Bulk with Cosmological Constant*, hep-th/9910219.
- 6 C. Csáki, M. Graessner, C. Kolda, and J. Terning, Phys. Lett. B426 (1999) 34.
- 7 J.M. Cline, C. Grosjean, and G. Servant, Phys. Rev. Lett. 83 (1999) 4245.

- 8 K. Koyama and J. Soda, *Birth of the Brane Universe*, gr-qc/0001033; J. Garriga and M. Sasaki, *Brane-World Creation and Black Holes*, hep-th/9912118; *Gravity in the Brane-World*, hep-th/9911055.
- 9 R.R. Caldwell, R. Dare, and P.J. Steinhardt, Phys. Rev. Lett. 80 (1998) 1582.
- 10 E. Di Pietro and J. Demaret, *Quintessence: Back to Basis*, gr-qc/9908071.
- 11 I. Zlatev, L. Wang and P.J. Steinhardt, Phys. Rev. Lett. 82 (1999) 896.
- 12 P.J. Steinhardt, in *Critical Problems in Physics*, edited by V.L. Fitch and D.R. Marlow (Princeton Univ. Press, Princeton, USA, 1997).
- 13 A. Masiero, M. Pietroni and E. Rocati, Phys. Rev. D61 (2000) 023504; K. Choi, *String or M Theory Axion as a Quintessence*, hep-th/9902292; P.Brax and J. Martin, *The Robustness of Quintessence*, astro-ph/9912046.
- 14 I.S. Gradshteyn and I.M. Ryzhik, *Table of Integrals, Series, and Products*, edited by A. Jeffrey (Academic Press, San Diego, USA, 1994).
- 15 M. Abramowitz and I. Stegun, *Handbook of Mathematical Functions* (Dover, New York, USA, 1964).
- 16 S. Perlmutter *et al.*, ApJ. 517 (1999) 565; A.G. Riess *et al.*, Astron. J. 116 (1998) 1009.
- 17 B.P. Schmidt *et al.*, ApJ. 509 (1998) 54.
- 18 S.W. Hawking and G.F.R. Ellis, *The Large Scale Structure of Space-Time* (Cambridge University Press, Cambridge, UK, 1973).
- 19 M. Visser, *Lorentzian Wormholes* (AIP, Woodbury, New York, USA, 1996).